Hadronic Matrix Elements and the Feynman-Hellmann Theorem

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Feynman-Hellmann Theorem

$$\frac{\partial E_\lambda}{\partial \lambda} = \left\langle \psi_\lambda \left| \frac{\partial \hat{H}_\lambda}{\partial \lambda} \right| \psi_\lambda \right\rangle$$

Relates matrix elements to variation in the spectrum

Matrix elements on lattice

$$\frac{\partial m_{\text{eff}}}{\partial \lambda} = \langle n | J | n \rangle$$

Construct analogous derivative to access lattice matrix elements

Effective mass

Lattice two-point correlation

$$C(t) = \sum_n Z_n^2 e^{-E_n t}$$

the ground state is approx.

$$m_{\text{eff}} = \frac{1}{\tau} \ln\left(\frac{C(t)}{C(t+\tau)}\right) \approx E_0$$

Effective mass derivative

$$R(t) \equiv \frac{\partial_\lambda C(t)}{C(t)}$$

$$\partial_\lambda C(t) = -\sum_{t'} \langle N(t) J(t') \bar{N}(0) \rangle$$

$$\left.\frac{\partial m_{\text{eff}}}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{\tau}\left[R(t) - R(t+\tau)\right]$$

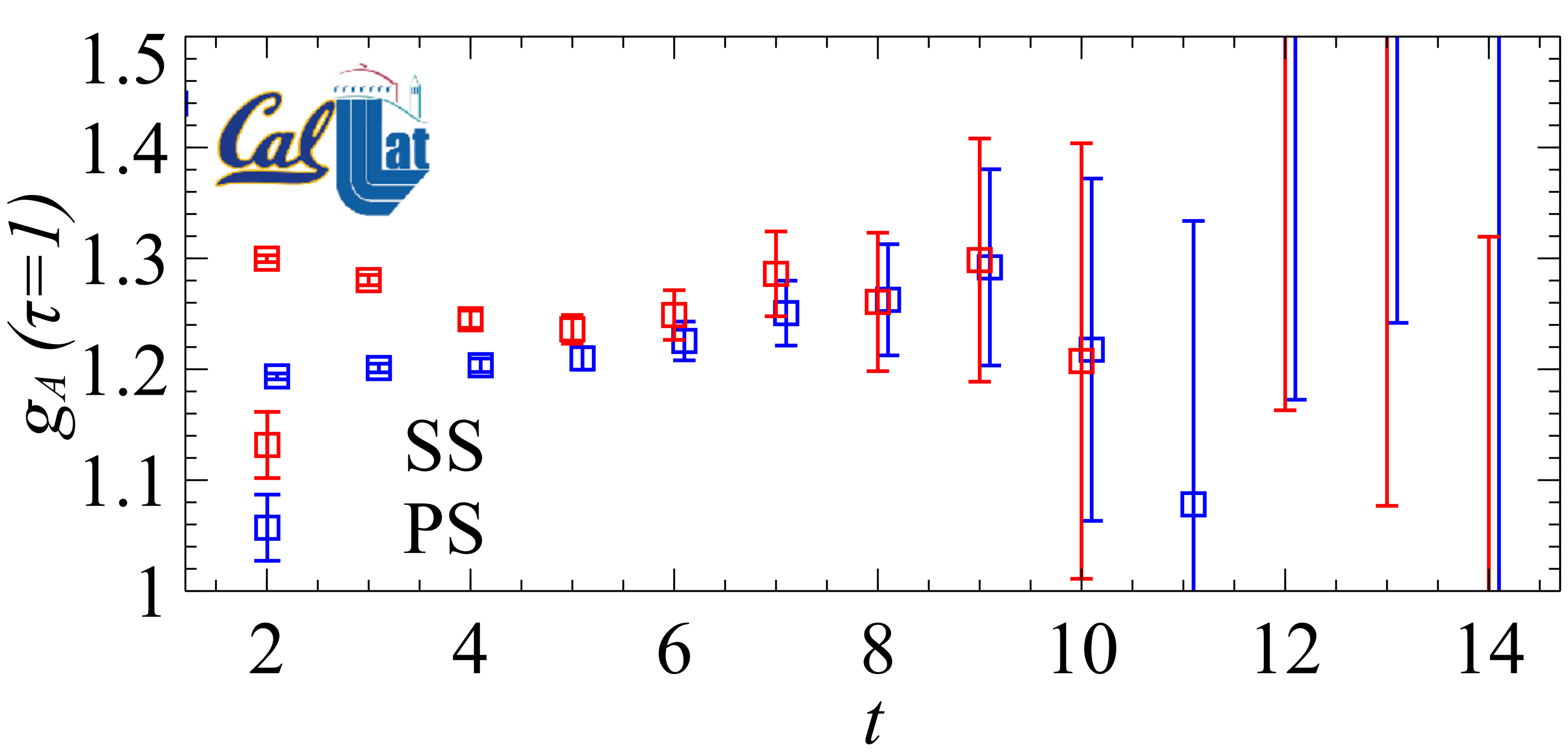
In the long time limit it the derivative of the effective mass is

$$R(t) \approx (t+1) g_A + \text{constant}$$

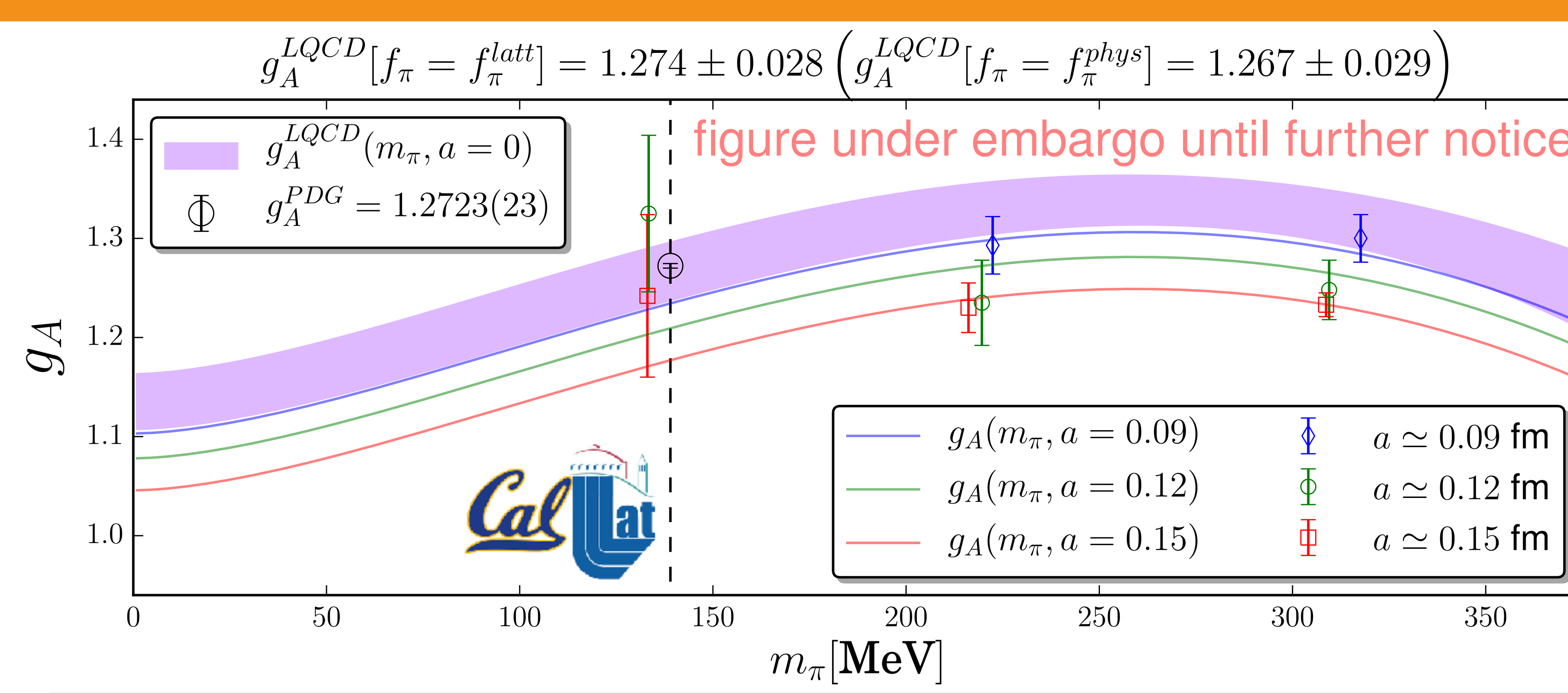
$$\frac{\partial m_{\text{eff}}}{\partial \lambda} \approx g_A + \mathrm{O}(e^{-E_n t})$$

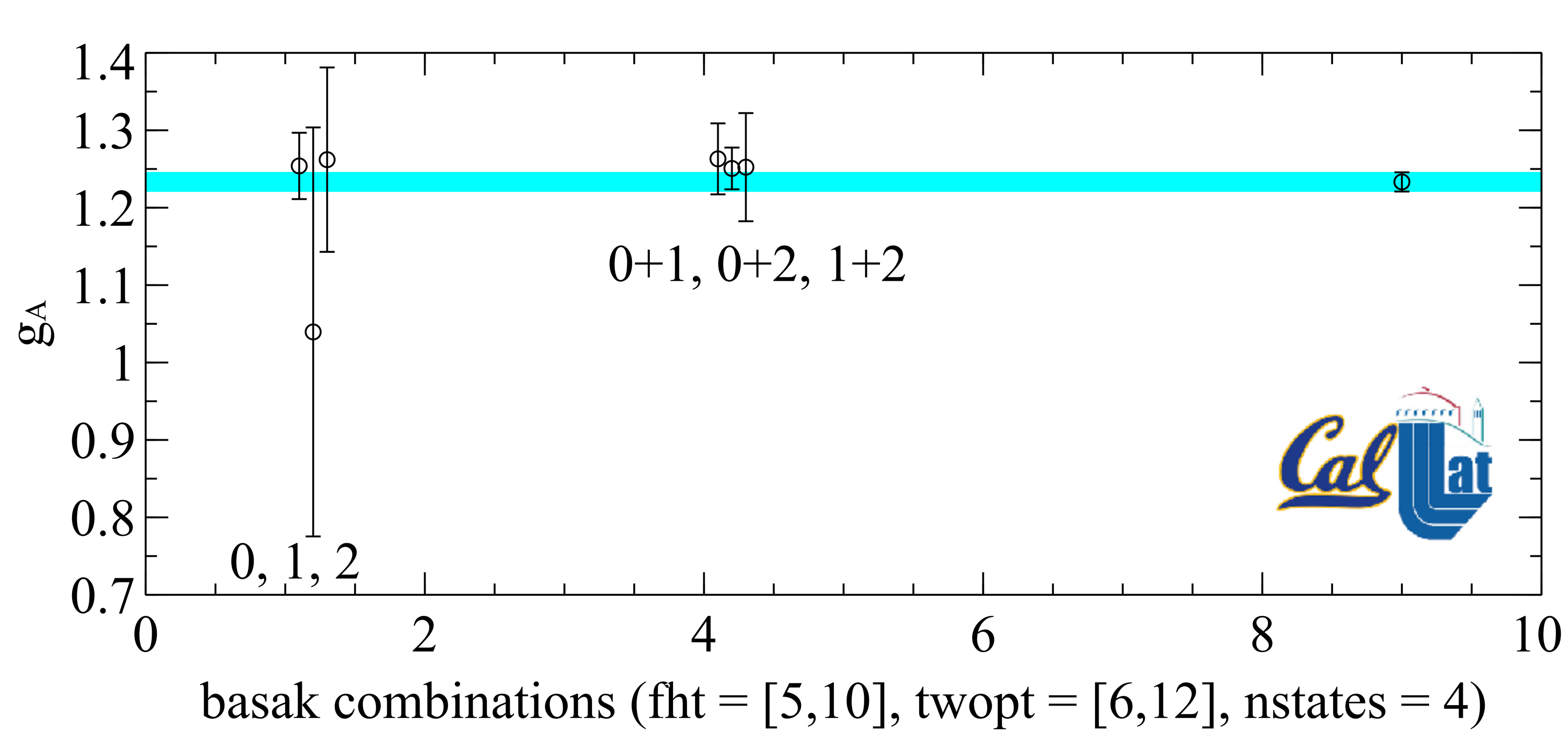
$$\text{where} \quad (E_n > E_0)$$

The CalLat effort



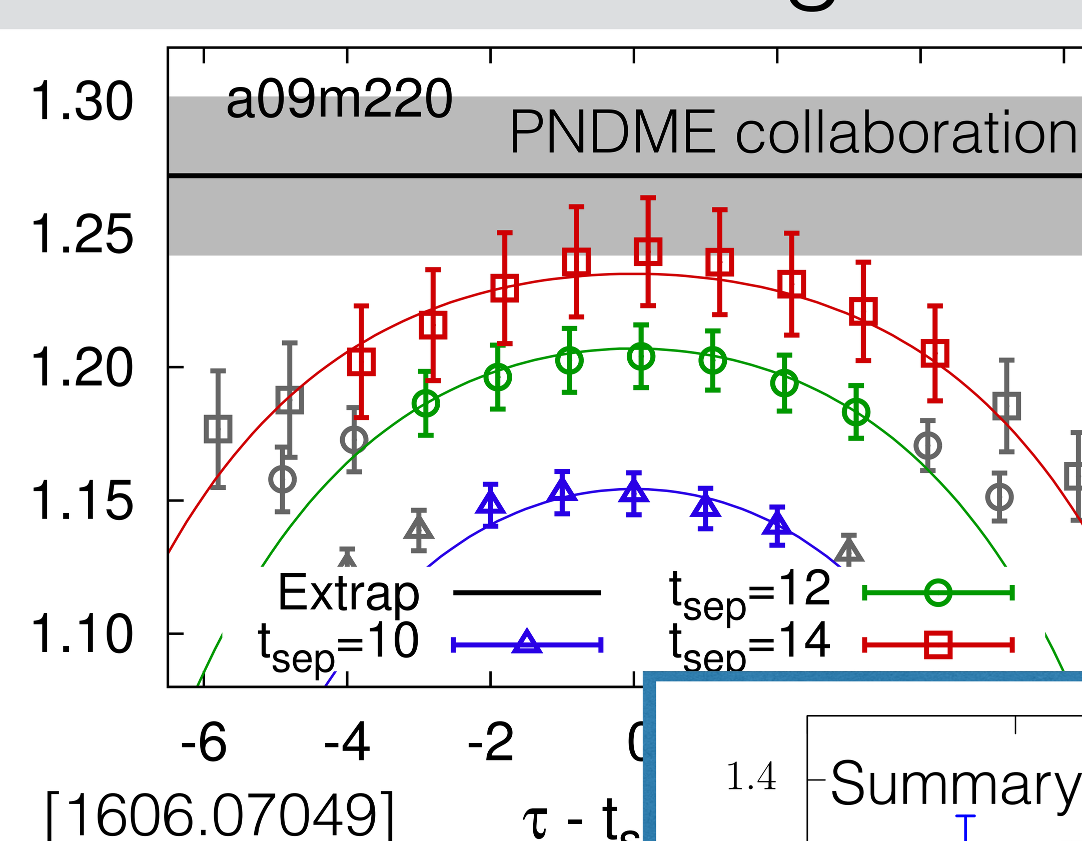
time dep. demonstrates ground state plateau at late time and exponential decay at early time





fit to multiple source/sink combinations improves statistics with negligible additional computation

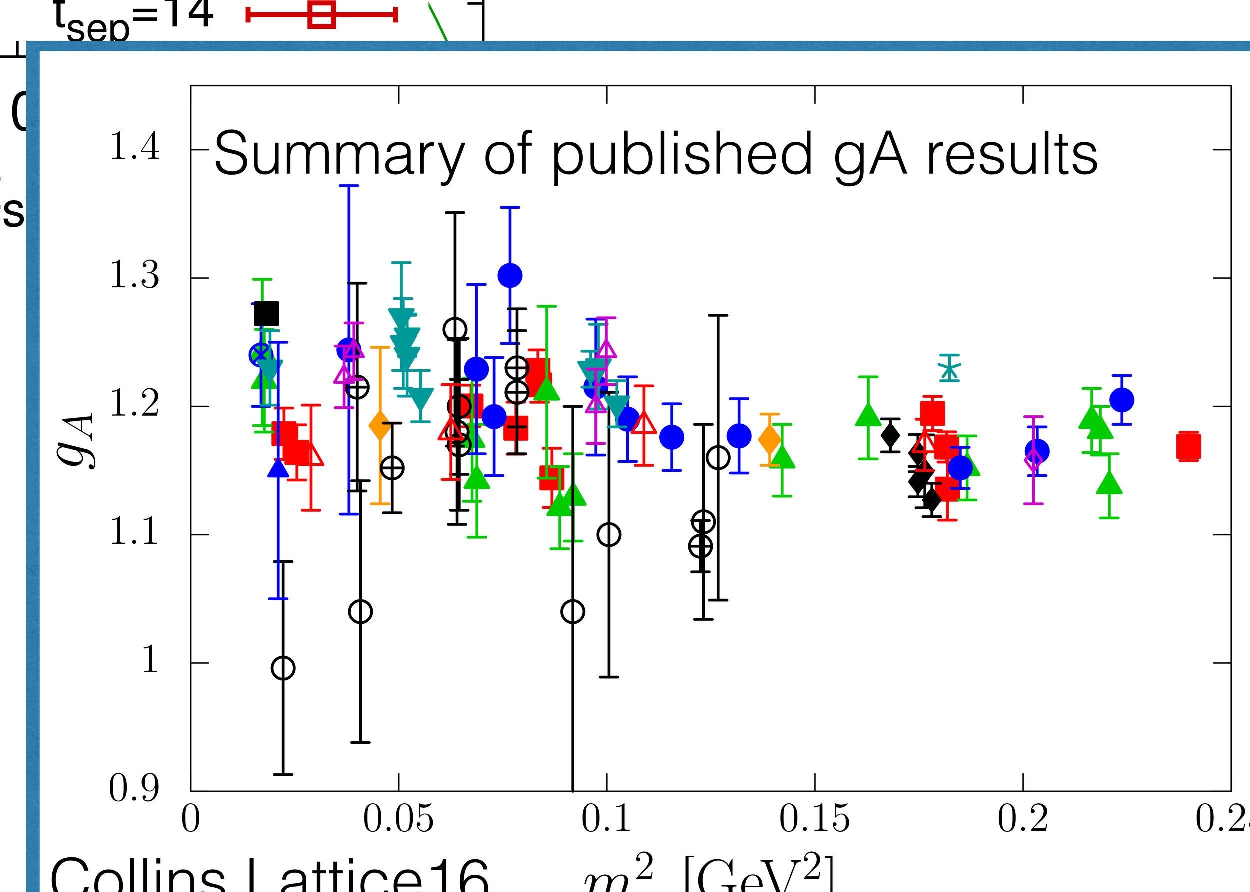
Status of gA on the lattice



time dependence from traditional method

ground state highly contaminated by excited states

published lattice results are systematically lower than exp. or have O(10%) uncertainty



Method summary

- time dependent systematics
- all time sep. for O(10) stat. improv.
- small time separation for **exponential** s/n improvement
- calculation cost equivalent to one time separation

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